

Exercises for Lectures 1 and 2

1. Random variable X satisfies Normal distribution with mean value 5 and variance 4. Find number “ a ” such that: (1) $P(X < a) = 0.9$; (2) $P(|X - 5| > a) = 0.01$

2. Random variable X satisfies:

X	0	1	2	3
P	0.729	0.243	0.027	0.001

- (1) Give the distribution function of X ; (2) Calculate $P(X \leq 0.5)$, $P(1 \leq X \leq 1.5)$.

3. Observe 45 rice blocks and count the number of *Tryporyza incertulas* (defined as X):

X	0	1	2	3	>3
No. of blocks	19	13	7	6	0

If X satisfies Poisson distribution, give the theoretical frequencies of X .

4. Random variable X satisfies Geometric distribution, i.e.

$$P(X = k) = pq^{k-1}, 0 < p < 1, q = 1 - p, k = 1, 2, \dots$$

Calculate $E(X)$.

5. X is a random variable, whose probability satisfies: $P(x=0)=1/10$, $P(x=1)=6/10$, $P(x=2)=3/10$, calculate $E(X)$.
6. Measure the diameter of the circle. The values follow the uniform distribution in $[a, b]$. Calculate the expectation of diameter and area of the circle.
7. Random variable X , whose probability distribution satisfies

X	-2	0	2
P	0.4	0.3	0.3

Calculate $V(X)$.

8. The temperature T of a city in February satisfies Normal distribution. The mean is -14.3°C and the standard variance is 3.5°C . Calculate $P(-15.4 \leq T < -13.2)$.
9. Shooting game: shoot for three times. The probability of hitting the target for the first shot is 0.8. If the first shot he hits the target, the second shot he also succeeds; otherwise, the probability of success for the second shot is 0.9. The third shot must be successful. Calculate the probability distribution of the number of shots which hit the target (X).
10. The cure rate of a new medicine is 0.8. Now 5 persons test the medicine. Calculate (1) the probability distribution of the number of cured persons. (2) The probability that at least 2

persons cured.

11. Random variable X satisfies Normal distribution with mean value 2 and variance 5. Random variable Y satisfies: $Y=5-2X$. Calculate $E(Y)$ and $V(Y)$.
12. X is a random variable which satisfies $P(X=k)=1/10$ ($k=2, 4, 6, \dots, 20$). Calculate $E(X)$ and $V(X)$.
13. There are two fields. The numbers of cotton plants in the two fields are the same. Count the numbers X and Y of *Helicoverpa armigera* in the two fields.

X	0	1	2	3
P	0.7	0.1	0.1	0.1

And

Y	0	1	2	3
P	0.5	0.3	0.2	0

Which field has more *Helicoverpa armigera*?

14. There are 100 seeds. 95 of them can germinate, and 5 cannot. Now select one seed randomly. Calculate the probability distribution of the number of germinated seeds.
15. X and Y are two random variables, and $V(X)=25$, $V(Y)=36$, $\rho_{XY}=0.4$. Calculate $V(X+Y)$ and $V(X-Y)$.
16. X and Y are two random variables whose values are:

X	1	2	3	4	5	6	7	8	9	10
Y	3	1	6	2	2	7	5	2	9	5

Calculate $\text{Cov}(X, Y)$ and ρ_{XY} .

17. Draw the bar-graph of a binomial distribution, e.g. $p=0.75$ and $n=10$
18. Draw in one graph the curves of four normal distributions, e.g. $N(0, 1)$, $N(0, 4)$, $N(3, 1)$, $N(3, 4)$
19. Given p.d.f. of the normal distribution $X \sim N(\mu, \sigma^2)$, $p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$,
show that $E(X)=\mu$ and $V(X)=\sigma^2$
20. What are the three major principles in experimental design? How could the three principles help the scientific studies?
21. What are RA Fisher's major contributions to statistics?